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Frequency Modulated OFDM (FM-OFDM): A Novel Waveform for Wireless Communication in Highly Doubly-Dispersive Channels

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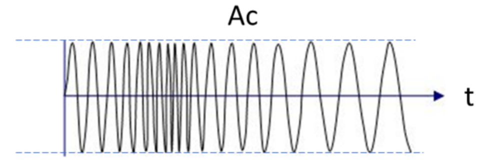
About me

- ***Javier Lorca Hernando*** **is doing research on signal processing topics for B5G & 6G at UC3M with Ana García Armada's group.**
- **He is also involved in 6G standards and pre-standards research activities for InterDigital, Wireless Labs Europe, UK. Previously he worked for Telefónica GCTIO.**
- **His research interests include, but are not limited to:**
    - Integrated sensing and communications.
    - Waveforms.
    - MIMO & RIS.
    - THz and Sub-THz communications.
    - Near-field.
    - Advanced channel estimation and equalization.

What is an FM-OFDM waveform?

- ***Constant-envelope waveform whose instantaneous frequency is an OFDM signal carrying the information* [1].**

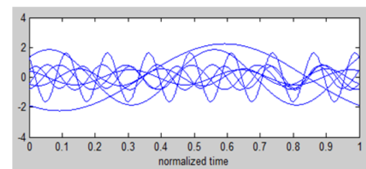


$$\bar{s}[n] = A_c \exp j\varphi_t[n]$$

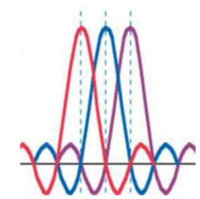
modulation index

$$f[n] = m \frac{1}{\sqrt{N_a}} \sum_{k=0}^{N-1} X[k] e^{\frac{j2\pi kn}{N}} \Rightarrow \varphi_t[n] = \varphi_0 + 2\pi \sum_{n'=0}^{n} f[n']$$

$f[n]$

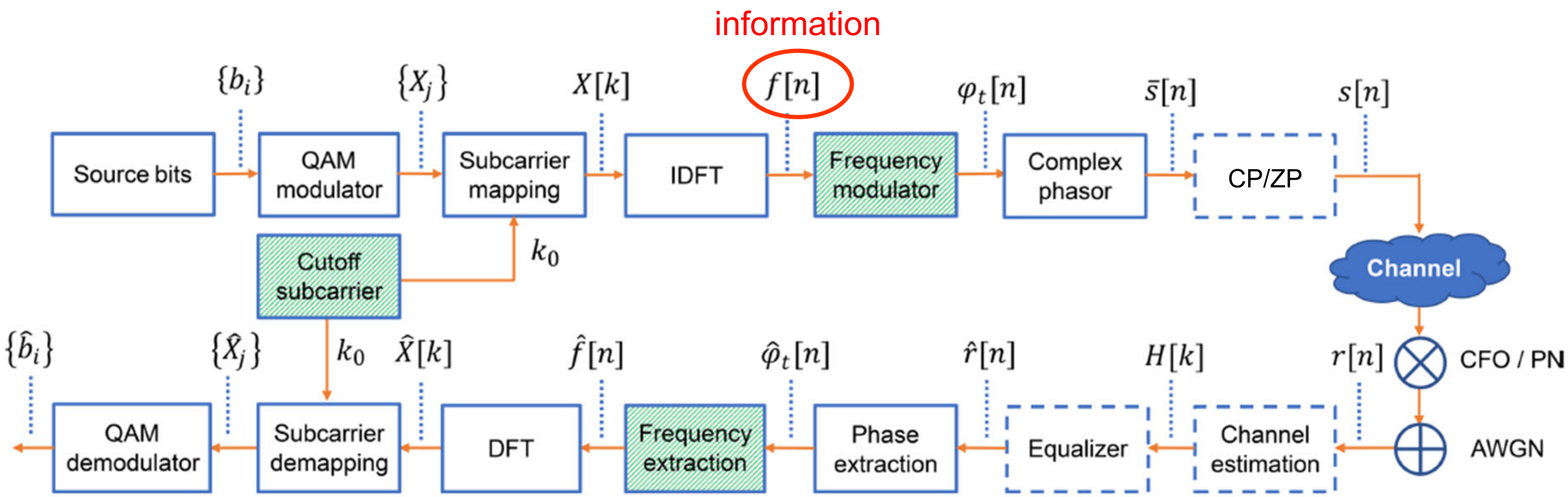


Spectrum of $f[n]$



- Information is mapped on the subcarriers in the <u>spectrum of the instantaneous frequency $f[n]$</u>.
- A parameter k_0 is introduced called the <u>cutoff subcarrier</u> such that *information is mapped above it in the instantaneous frequency spectrum.*
- *This is done to limit the impact of Doppler, phase noise (PN), and carrier frequency offset (CFO) impairments on the information-bearing subcarriers.*

FM-OFDM waveform generation



- **Shaded blocks are additional to CP-OFDM processing blocks.**
- **Dashed blocks are not required in AWGN or flat-fading conditions.**
- **ZP (Zero Padding) is more convenient in doubly-dispersive channels, whereas CP (Cyclic Prefix) can be valid at moderate Doppler/PN regimes.**

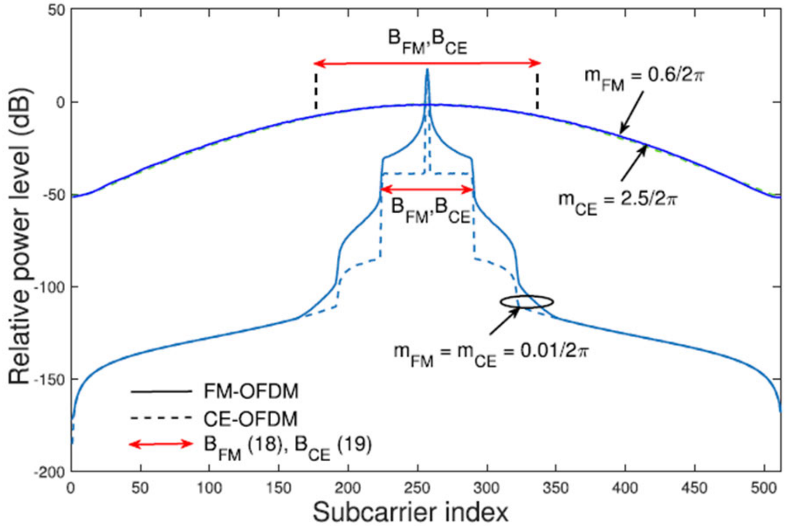
Rationale for frequency modulation with an OFDM signal

- ***Phase/frequency impairments are additive in the instantaneous frequency. No ICI appears as a result.***
- ***Encoding the information in the phase differences (rather than in the phases, like in CE-OFDM* [2]*) brings added robustness to phase/frequency impairments.***
  - Frequency tends to exhibit much slower variations than phase.
  - Both Doppler and PN concentrate their effects on the lower subcarriers of the instantaneous frequency (CFO has no contents beyond DC!).
- **Modulating with an OFDM signal allows further control:**
  - Information can be mapped on subcarriers above $k_0 \gtrsim \left\lfloor \frac{\max(f_D, W_{PN})}{SCS} \right\rfloor$.

Bandwidth and Spectral Efficiency

◆ RMS Bandwidth:

- $B_{rms} \simeq 2\left(mf_s + \frac{N_a}{2T_s}\right)$ (Carson's rule).
- N_a: no. active subcarriers; T_s: symbol period; f_s: sampling frequency.
- *Similar to CE-OFDM, but with slightly different modulation indexes.*



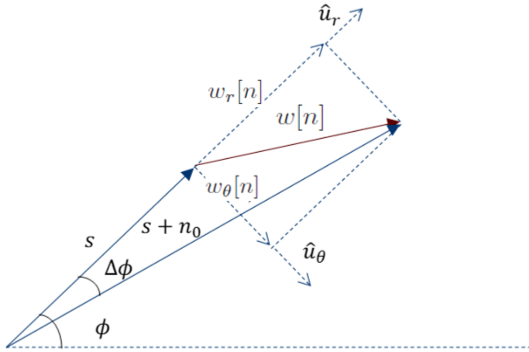
◆ Spectral efficiency: $\epsilon = \frac{N_a K_{QAM}}{4T_s\left(mf_s + \frac{N_a}{2T_s}\right)}$

- At very low m, $\epsilon \to \frac{K_{QAM}}{2}$ from the hermiticity of the instantaneous frequency spectrum: only half of the subcarriers bear information.
  - ✓ This can be improved by using offset modulation techniques [8] or subcarrier-dependent power allocation schemes [9].

SNR in AWGN conditions with no channel/PN impairments

- **SNR of the *k*-th subcarrier:**

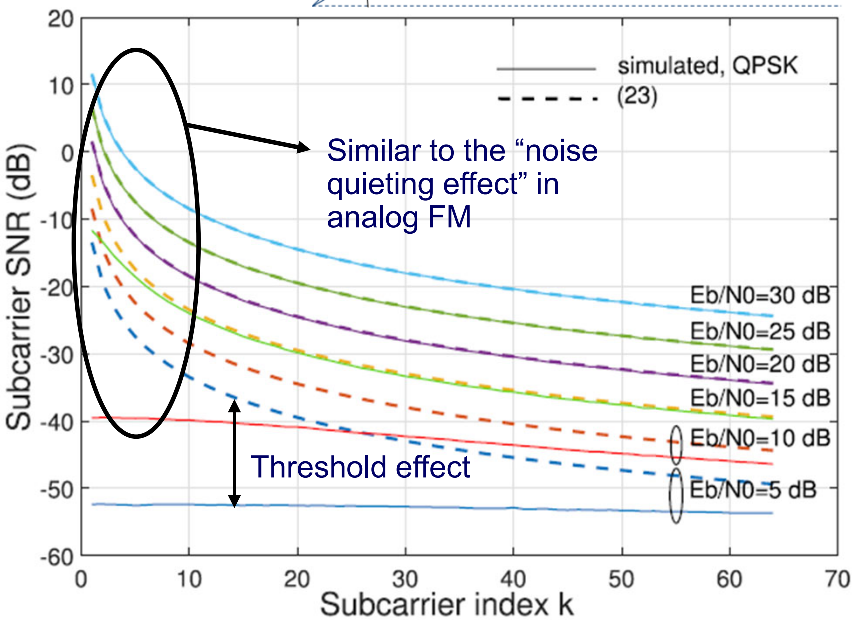
$$SNR_k^{FM} = \frac{2\pi^2 m^2 N}{N_a \mathbb{E}\left\{\arctan^2 w'[n]\right\} \underbrace{\left(1-\cos\frac{2\pi k}{N}\right)}_{\text{Noise reduction at the lower subcarriers}}}, \quad w'[n] \triangleq \frac{w_\theta[n]}{A_c + w_r[n]}$$



  - When SNR ≥ threshold SNR (typically, 10 dB):

$$SNR_k^{FM} \simeq \frac{8\pi^2 m^2 N}{N_a\left(1-\cos\frac{2\pi k}{N}\right)} SNR_{in}, \qquad (23)$$

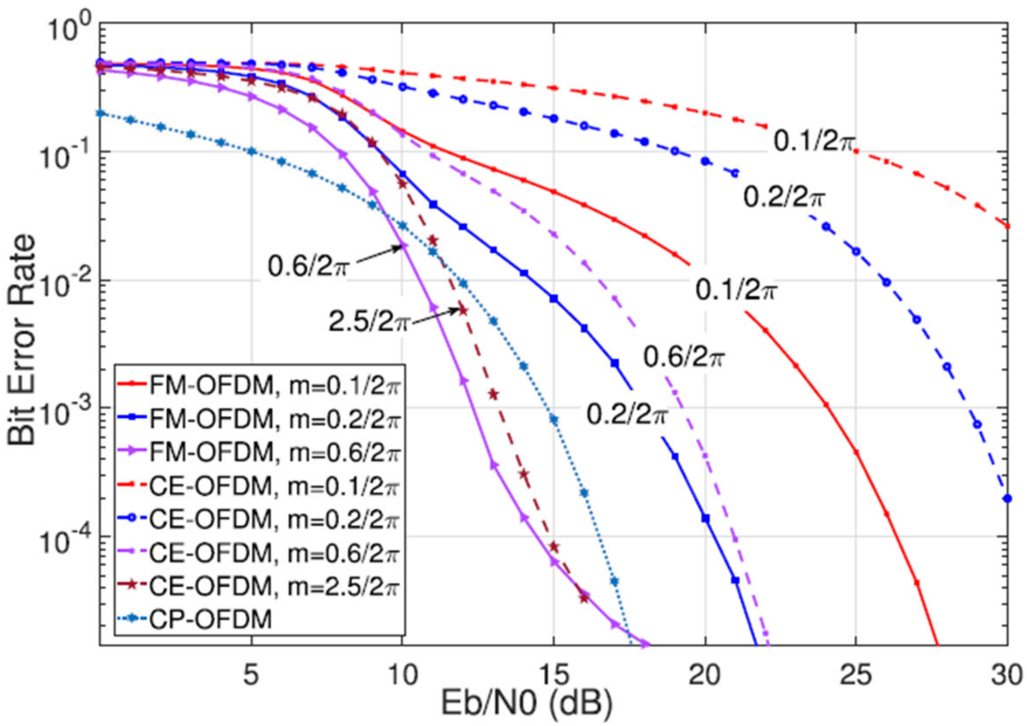
- **Below the threshold SNR, errors can be significant when phases cross the boundaries from $\pm\pi$ to $\mp\pi$.**
  - Threshold extension techniques can reduce the threshold SNR below 10 dB [2,3].
- **Phase errors also pose a limit to m as a function of the modulation order and the channel.**



BER in AWGN conditions with no channel/PN impairments

- **Modulation index brings extra performance control at the cost of varying BW.**
  - Depends on the channel, modulation order, and no. active subcarriers.
- **At high *m*, BER outperforms CP-OFDM above the threshold.**
  - Typically, 10 dB (can be reduced via threshold extension techniques [3]).
- ***FM-OFDM and CE-OFDM are equivalent in terms of BW, performance, and spectral efficiency in AWGN.***
  - max. $m_{FM} = 0$ to $\sim 0.6/2\pi$
  - max. $m_{CE} = 0$ to $\sim 2.5/2\pi$

AWGN, 64QAM, $N = 512$, $N_a = 64$



Analysis of FM-OFDM in flat-fading channels (I)

- **Received signal (ignoring CP/ZP):**

$$r[n] = b[n]\bar{s}[n] \exp j\left(\varphi_e[n] + \psi\right) + w[n],$$

  - $b[n], \psi$: channel's time-varying amplitude and phase, respectively.
  - $\varphi_e[n] = \varphi_D[n] + \varphi_P[n] + \varphi_C[n]$ contains the time-varying phases of Doppler, PN and CFO impairments,
  - $w[n] = w_r[n] + w_\theta[n]$: circularly-symmetric Gaussian term $\sim\mathcal{CN}(0, N_0)$

- **Taking the phase differences, assuming slow variations:**

$$\frac{1}{2\pi}\nabla \arg r[n] = f[n] + \frac{1}{2\pi}\nabla\left(\varphi_e[n] + \Delta\varphi_t[n]\right), \quad \Delta\varphi_t[n] = \arctan\frac{w_\theta[n]}{A_c + w_r[n]}.$$

⇒ *Channel state has no impact on detection. Channel estimation and equalization are no longer required to recover the signal.*

⇒ *Impairments are additive in the instantaneous frequency domain.*

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Analysis of FM-OFDM in flat-fading channels (II)

- **What is the spectral width W_e of the Doppler, PN, and CFO impairments when seen in the instantaneous frequency domain?**
  - Combined Doppler and PN spectral width: $W_e \lesssim \max(f_D, W_{PN})$.
    - ✓ f_D: Doppler spread; W_{PN}: PN spectral width.
  - CFO spectral width: 0 (only involves a DC component!).
- **Impairments from Doppler, PN and CFO can be confined within a spectrum region roughly bounded by:**

$$k_0 \gtrsim \left\lfloor \frac{\max(f_D, W_{PN})}{SCS} \right\rfloor.$$

➡ Just a <u>first estimation</u>: the optimum k_0 should be assessed by simulation.

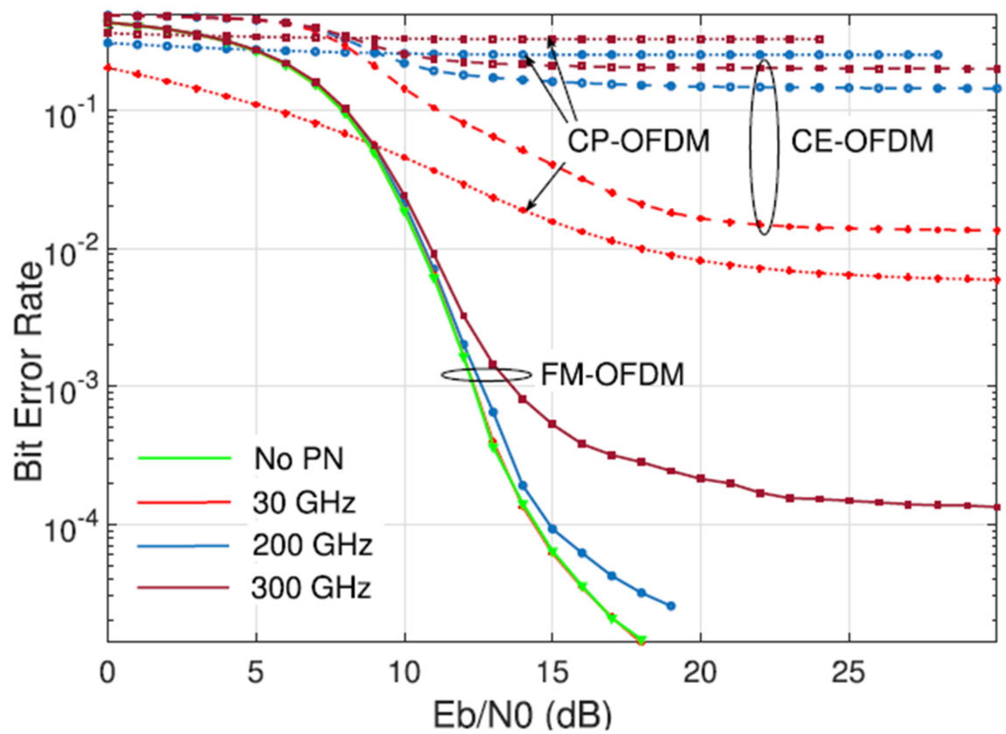
- ✓ Depends on the modulation order and BER operating point.

➡ $k_0 = 0$ in most practical *underspread* channels (where delay spread is much more relevant than doppler spread), including NTN channels (see sl. 13-14).

BER in AWGN conditions under PN with $k_0 = 0$

- **Plots obtained without PN compensation.**
- **No channel estimation or equalization in FM-OFDM.**
  - Much higher resilience than MMSE-detected CP/CE-OFDM above the threshold SNR.
  - Barely affected by PN up to at least 200 GHz.
- **CP-OFDM and CE-OFDM are barely usable above 30 GHz.**
  - Need to mitigate PN-induced CPE (Common Phase Error) and ICI.

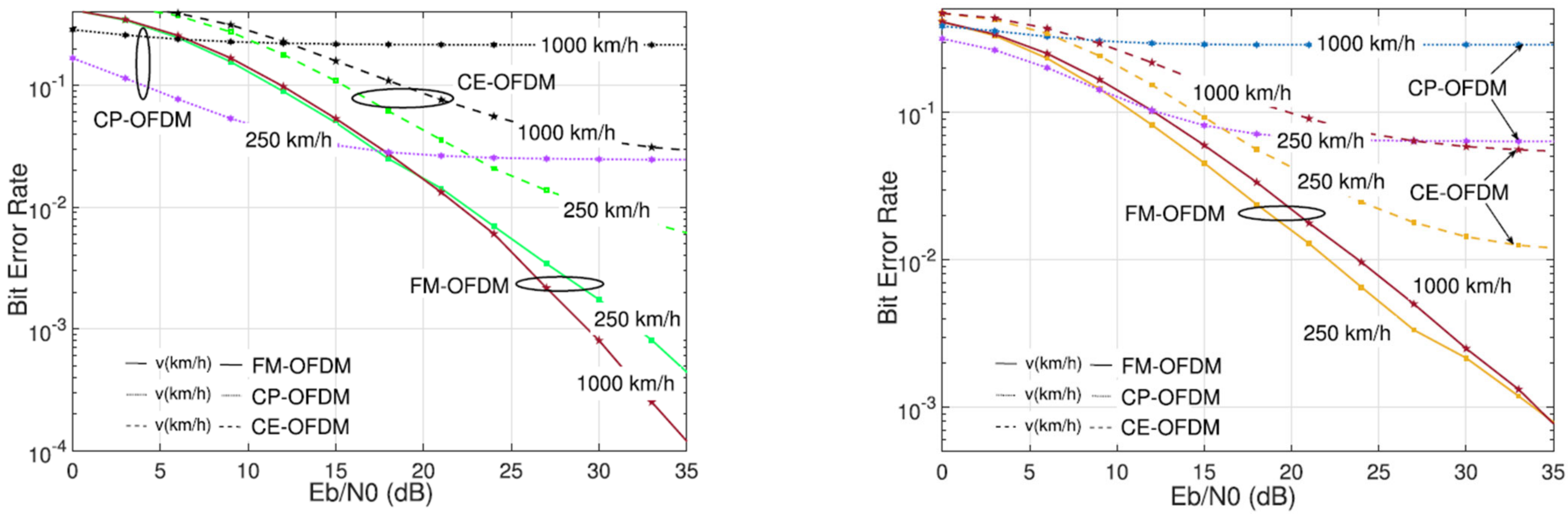
AWGN, 64QAM, $k_0 = 0$, $m = 0.6/2\pi$, $N = 512$, $N_a = 128$



BER in flat-fading channels under high Doppler with $k_0 = 0$

Flat Rayleigh, QPSK, $k_0 = 0$, $m = 0.6/2\pi$, $N = 512$, $N_a = 128$

Flat Rayleigh, 16QAM, $k_0 = 0$, $m = 0.6/2\pi$, $N = 512$, $N_a = 128$



- **No channel estimation or equalization needed in FM-OFDM**
  - In QPSK, mobility is even beneficial from the extra Doppler diversity:
    - ✓ *Any loss from a deep fade in part of the symbol may be compensated by samples in another part of the symbol with less deep fade contributing to the same subcarrier.*
  - In 16QAM, degradation from 250 to 1000 km/h is lower than 2 dB.
  - Further characterization of FM-OFDM in NTN channels is provided in [5].

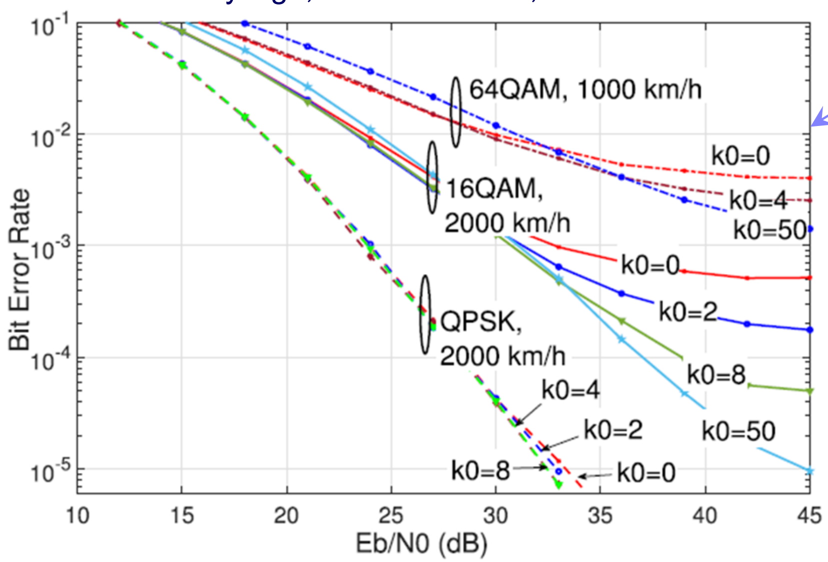
Impact of k_0 in NTN flat-fading scenarios (I)

- **Estimated k_0 is ~0 in most practical cases.**
- **Numerical results shows the need for $k_0 > 0$ only at very high speeds and/or small SCS.**
  - Only at ≥ 1000 km/h, especially at LEO speeds (25000 km/h).
  - Increasing k_0 beyond the estimated value brings diminishing returns and, in some cases, degrades BER (from the 1-cos() subcarrier noise, sl. 7).
- **The optimum k_0 is a trade-off between Doppler/PN avoidance and SNR degradation (see next slide).**

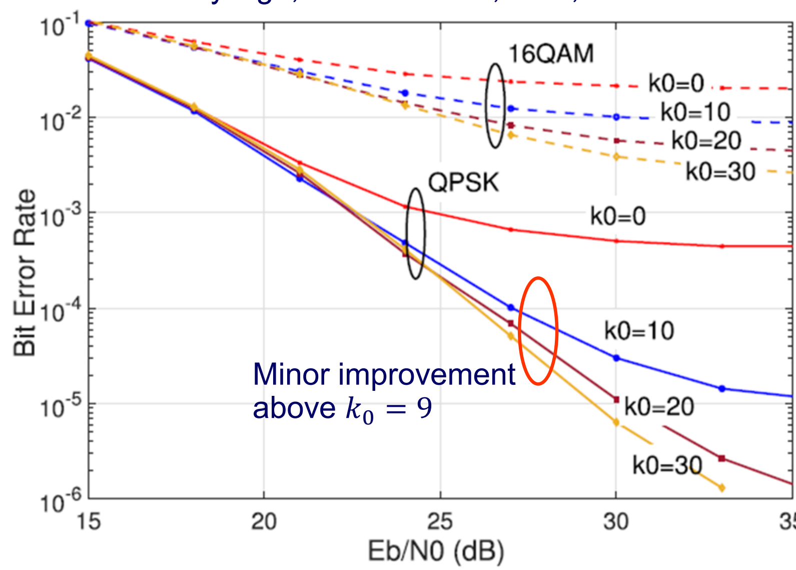
Estimated cutoff subcarrier with no PN:

f_c (GHz)	SCS (kHz)	v (km/h)	f_D (kHz)	k_0
6	15	500	2.77	0
		1000	5.55	0
		2000	11.11	0
		25000	138.88	9
6	3.75	500	2.77	0
		1000	5.55	1
		2000	11.11	2
		5000	27.77	7

Flat Rayleigh, $SCS = 3.75\ kHz$, 1000/2000 km/h

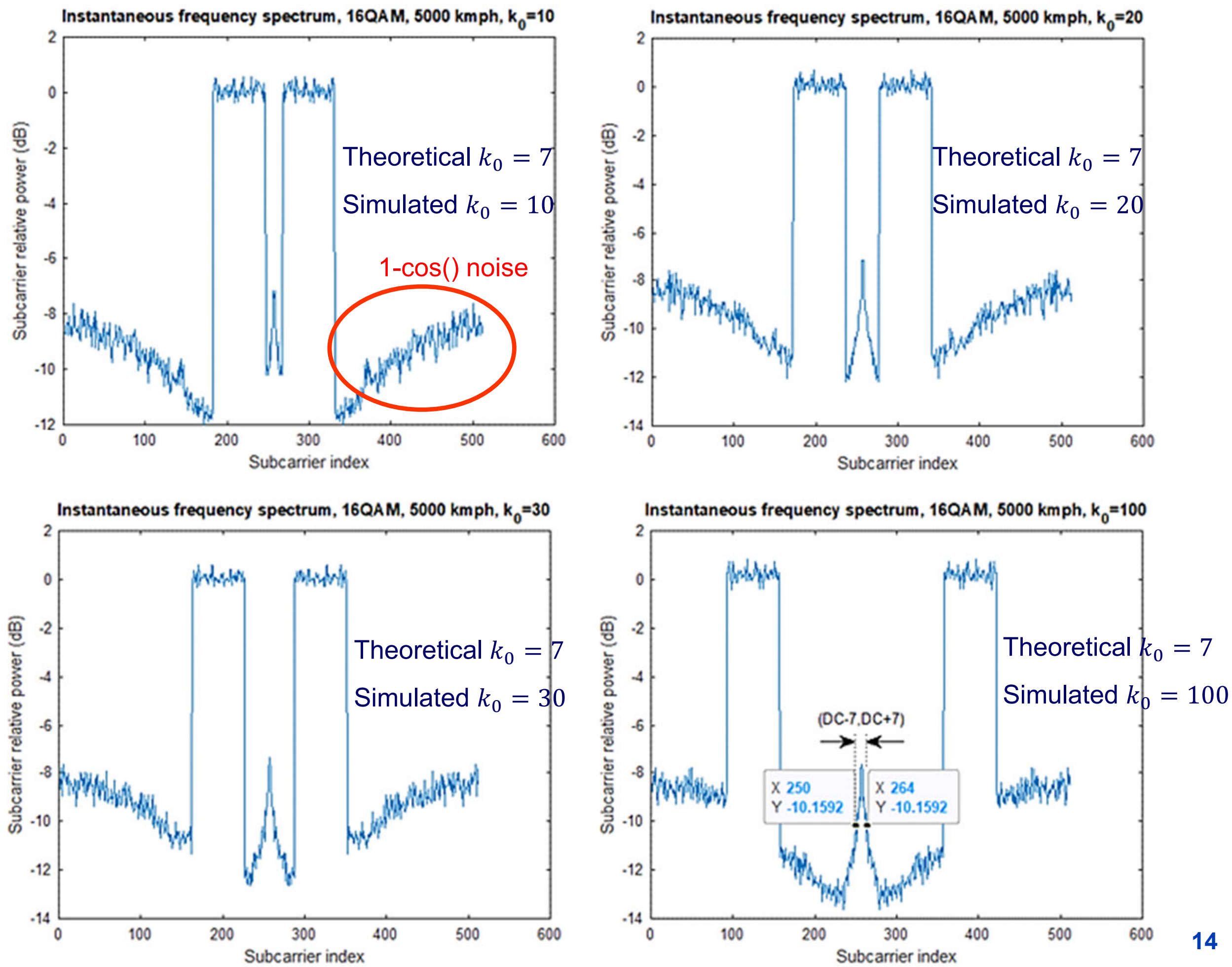


Flat Rayleigh, $SCS = 15\ kHz$, LEO, 25000 km/h



Impact of k_0 in NTN flat-fading scenarios (II)

- **How is Doppler seen in the instantaneous frequency spectrum?**
  - Doppler impairments are captured in the central subcarriers.
  - Increasing k_0 has a positive effect until the 1-cos() noise starts to degrade SNR.
- **Optimum k_0 should be assessed via simulation.**
  - The value that yields best performance depends on the modulation and the *spillover of Doppler and phase noise* over the data subcarriers.



Analysis of FM-OFDM in frequency-selective channels (I)

- **Equalization is needed to overcome the channel's delay spread.**
- **However, it is generally not capable of perfectly aligning multipath components, and residual signal contributions remain after equalization.**
- **The instantaneous frequency at the output of an equalizer reads:**

$$\frac{1}{2\pi}\nabla \arg \hat{r}[n] = f[n] + \frac{1}{2\pi}\nabla\left(\hat{\varphi}_e[n] + \hat{\psi} + \hat{\epsilon}[n] + \Delta\varphi_t[n]\right)$$

  - $\hat{\psi}$: residual (constant) phase.
  - $\hat{\varphi}_e[n]$**:** residual (time-varying) phase from Doppler, PN, and CFO.
  - $\hat{\epsilon}[n] = \arg\left(\sum_{z=0}^{N-1} \hat{h}_e[n,z]\bar{s}[z]\right)$: residual signal term stemming from imperfect equalization, where $\hat{h}_e[n,z]$ is a residual impulse response.
- **Since $\hat{\epsilon}[n]$ contains a convolution of the signal and a residual channel, the spectral width of the impairments obeys $W_e \geq f_D$. Then:**

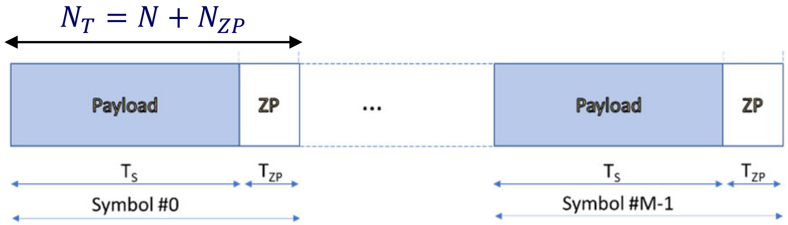
$$k_0 \gtrsim \left\lceil \frac{\max(W_e, W_{PN})}{SCS} \right\rceil .$$

Analysis of FM-OFDM in frequency-selective channels (II)

- **The extent to which multipath is removed from the received signal determines performance.**
  - The more effectively the equalized signal resembles the output of a one-tap fading channel, the closer W_e will be to f_D.
  - *Equalization performance is determined by the amount of residual signal contribution whose phase variations $\hat{\epsilon}[n]$ are not absorbed by the cutoff subcarrier*.
- ***The goal of the equalizer is to effectively remove delay spread from the received signal.***

➡ *It must combine the time-varying multipath responses into a single tap response within the symbol but <u>does not need to estimate Doppler components as long as they mostly remain below the cutoff subcarrier</u>.*

Piecewise equalization with Zero-Padded waveforms (I)



$$\mathbf{H} = \begin{bmatrix} b_0[0] & 0 & 0 & \cdots & b_1[0] \\ b_1[1] & \ddots & \vdots & & \vdots \\ \vdots & \cdots & 0 & \cdots & b_{L-1}[L-2] \\ b_{L-1}[L-1] & \cdots & 0 & \cdots & 0 \\ 0 & \ddots & b_0[N_T-L] & \cdots & \vdots \\ \vdots & \cdots & b_1[N_T-L+1] & \ddots & \vdots \\ \vdots & & \vdots & & 0 \\ 0 & \cdots & b_{L-1}[N_T-1] & \cdots & b_0[N_T-1] \end{bmatrix}$$

- **Zero-Padding brings zeros at the symbols' inputs because of the circular convolution. Thus, the L top-right coefficients in the CIR matrix can be safely set to 0 [4].**
- **The resulting CIR channel matrix $\boldsymbol{H} \in \mathbb{C}^{N_T \times N_T}$ becomes *upper triangular*:**
  - The output signal reduces to a linear combination of $L-1$ previous samples weighed by the channel coefficients.
  - The LTV channel can therefore be locally described by a *piecewise-approximated LTI channel whose CIR matrix $\boldsymbol{H}_i \in \mathbb{C}^{N_T \times N_T}$ corresponding to the i-th interval $i\Delta \leq n \leq (i+1)\Delta$ is circulant.*

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Piecewise equalization with Zero-Padded waveforms (II)

- **A set of N_L piecewise channel responses are obtained that locally characterize the LTV channel at the N_L intervals:**

$$h[n,z] \simeq h_i[z], \forall n \in [0, N-1]: \; i\Delta \leq n < (i+1)\Delta,$$

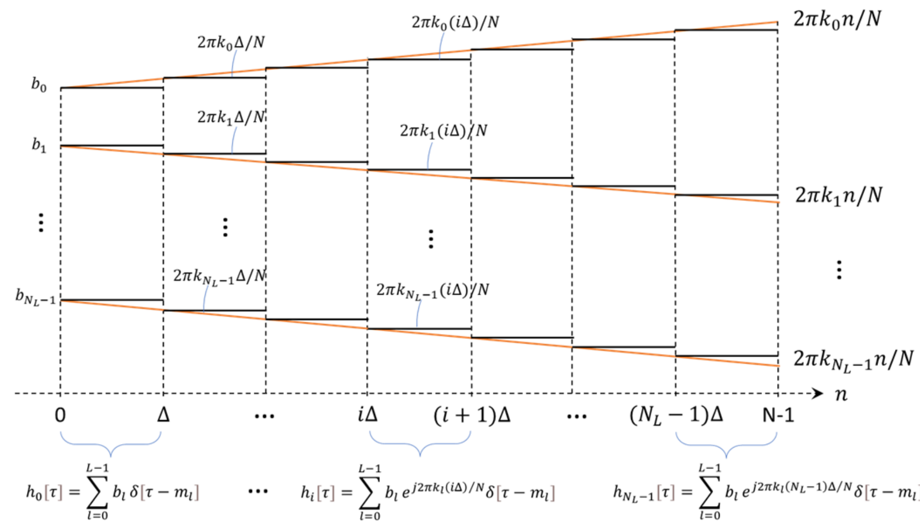
$$h_i[z] \triangleq h[(2i+1)\Delta/2, z] = \sum_{l=0}^{L-1} b_{i,l}\delta[z - z_l]$$

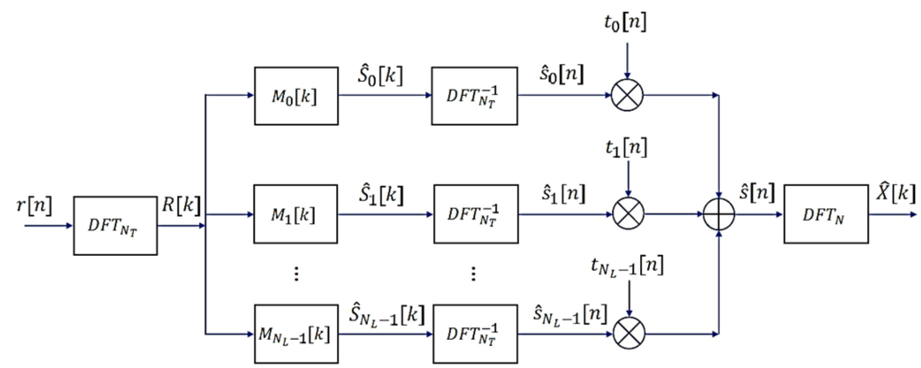
- **This suggests a strategy based on a bank of equalizers for each *i*-th interval, $0 \leq i \leq N_L - 1$:**

$$\mathbf{R} \simeq \mathbf{H_i S}, \; i \in [0, N_L - 1]$$

- **Outputs can then be combined via window functions $t_i[n]$ (raised cosine, trapezoidal, etc.) to yield the equalized output:**

$$\hat{s}[n] = \sum_{i=0}^{N_L-1} t_i[n] \mathrm{DFT}_{N_T}^{-1}\left\{M_i[k]R[k]\right\}.$$

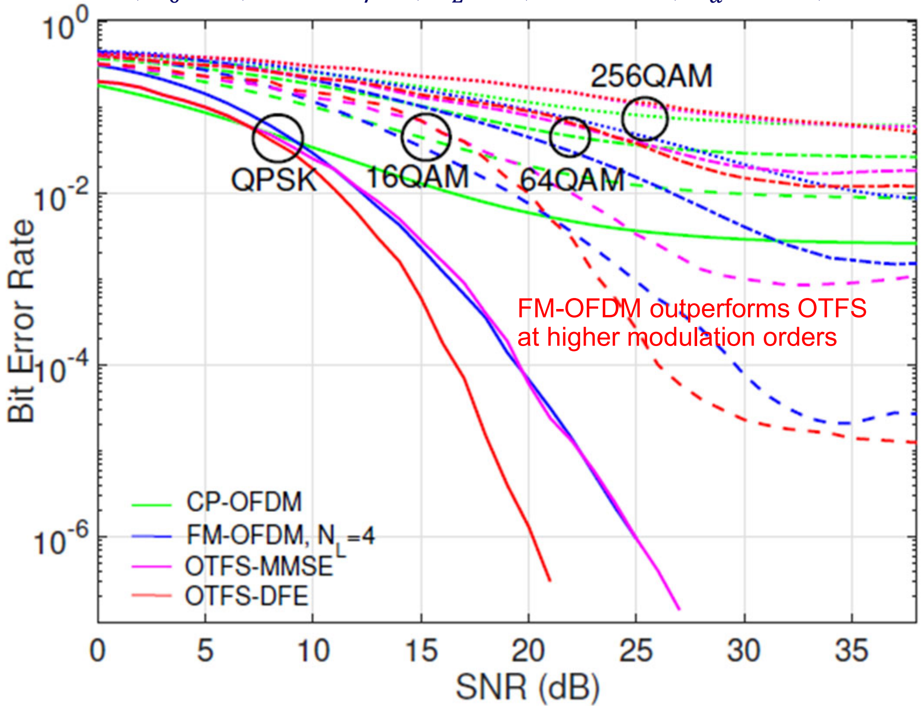




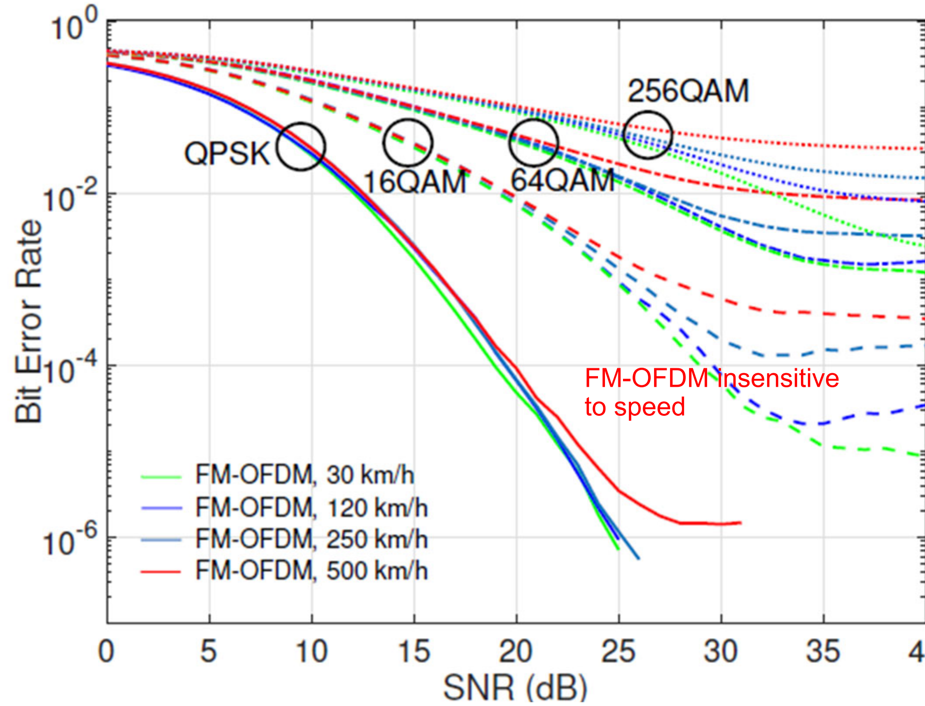
BER in highly doubly-dispersive channels w/ piecewise equalization

- **Results of piecewise-equalized FM-OFDM (further elaborated in [6]):**
  - Uncoded BER @120 km/h: comparable to MMSE-OTFS in QPSK, superior to MMSE-OTFS in 16QAM, superior to DFE-OTFS in 64/256QAM above the threshold SNR.
  - Rather insensitive to speed up to 500 km/h in QPSK and 16QAM.
  - Remarkably good performance at high modulation orders (16/64/256QAM).

TDL-C, $k_0 = 0$, $m = 0.8/2\pi$, $N_L = 4$, $N = 1024$, $N_a = 600$, 120 km/h



TDL-C, $k_0 = 0$, $m = 0.8/2\pi$, $N_L = 4$, $N = 1024$, $N_a = 600$, 30-500 km/h



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Conclusions and further lines of work

- **FM-OFDM can cope with highly dispersive channels thanks to:**
  - Strong resilience to time-varying impairments thanks to the *differential phases* and the presence of the *cutoff subcarrier*.
  - *No estimation or equalization needed in frequency-flat channels* (or Rician with a high K factor) regardless of the Doppler and PN severity.
  - The equalizer only needs to compensate the time-varying multipath profile in frequency-selective channels. *No need to estimate any Doppler components.*
- **Further lines of work:**
  - Channel estimation and equalization (ongoing work at UC3M [6][7]).
  - Receiver structures for reduction of the threshold SNR.
  - Inclusion of MIMO capabilities.
  - Spectral shaping to improve spectral confinement.
  - Diversity order and comparison with state-of-the-art (e.g., OTFS, AFDM).

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